

COMPONENTS OF CHANGE IN A PROPORTION

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The purpose of this paper is to describe a means of partitioning the difference between a proportion or a percentage observed at two time periods into separate components of change. I will begin with a simple case explained through an example, and then develop an elaboration having a particular application to migration research.

I first became interested in this problem some time ago when doing research on part-time farming. The proportion, or percentage, of farmers who work more than 100 days off the farm (a common definition of "part-time") has steadily increased in the U.S. since this information first became available with the 1930 Census of Agriculture. During this same period the total number of farmers has declined considerably almost everywhere in the nation. Consequently, an increase in the proportion of farmers working part-time between two censuses could be due in part both to an increase in the number of part-time farmers and to a decrease in the number of full-time farmers, or possibly to a decline in the number of part-time farmers more than offset by the decline in the number of full-time farmers. Of course, the number of full-time farmers could also be increasing in an area (though this would be unusual) with part-time farmers either increasing or decreasing, and the change in the proportion part-time would again depend upon the magnitude of the component changes.

We wish a means, then, to allocate the amount of change in the proportion part-time to these two components. This problem parallels to some extent the work of Kitagawa (1955) on components of a difference between two rates. The

basic terms utilized here are simple proportions rather than rates conceived as weighted averages, however. The approach here borrows from the logic and notation of factorial analysis of variance, although the solution is not an analysis of variance. (See Kempthorne, 1952: 234-248; Namboodiri, Carter and Blalock, 1975: 356-385.)

Two Components of Change

Consider first the 2^2 factorial design, that is, an experiment having two factors, each with two levels, presence or absence, with the letter a indicating presence of the first factor, and b presence of the second factor. Then the four outcomes, or treatment combinations, are ab (Presence of a and b); a (presence of a only); b (presence of b only); and 1 (absence of both factors). The effect of the first factor is thus:

$$A = 1/2[(ab - b) + (a - 1)] \quad (1)$$

or the average of the difference between the outcome with a present and a absent when b is present, and the outcome with a present and a absent when b is absent. Similarly the effect of the second factor is

$$B = 1/2[(ab - a) + (b - 1)]. \quad (2)$$

Following this development, an interaction term, AB, may be defined as the difference between the a effect with b present and with b absent, or identically, the difference between b effect with a present and with a absent.

$$AB = 1/2 [(ab - b) - (a - 1)] = 1/2 [(ab - a) - (b - 1)]. \quad (3)$$

Now, in our particular problem, consider the change in a proportion over time, where the initial numerator is X and the initial denominator $X + Y$. Then if the algebraic increment to X between time 1 and time 2 is represented Δa , and the algebraic increment to Y between time 1 and time 2 is Δb , the change in the proportion is

$$\frac{X + \Delta\alpha}{X + Y + \Delta\alpha + \Delta b} - \frac{X}{X + Y} \quad (4)$$

The outcome with a and b present is ab $=$ $\frac{X + \Delta\alpha}{X + Y + \Delta\alpha + \Delta b}$

The outcome with a only present is a $=$ $\frac{X + \Delta\alpha}{X + Y + \Delta\alpha}$

The outcome with b only present is b $=$ $\frac{X}{X + Y + \Delta b}$

The outcome with neither a nor b is 1 $=$ $\frac{X}{X + Y}$

The A effect, B effect and AB interaction terms may be found by substituting values for ab , a , b and 1 in the formulas (1), (2), (3) above. Note that the difference between proportions (change in the proportion over time) is equal to $ab - 1$. Next it can be shown that $A + B = ab - 1$, thus partitioning the change in proportion into two effects.

$$\begin{aligned} A + B &= 1/2[(ab - b) + (a - 1) + (ab - a) + (b - 1)] \\ &= 1/2[ab - b + a - 1 + ab - a + b - 1] \\ &= ab - 1 \end{aligned}$$

A is the effect of the change in the numerator over time, and B is the effect of change in the complement to the numerator over time, where the numerator plus the complement is the denominator.

The part-time farming example may be expressed logically by a 2^2 problem as in Table 1. Each treatment combination is what the proportion part-time would be under the circumstances indicated.

Data from the 1959 and 1969 Censuses of Agriculture are used to demonstrate this method. The number of farmers of both types is given for Alabama and for Illinois in Table 2.

The treatment combinations are, for Alabama:

$$\begin{aligned}
 ab &= \frac{40,595 - 5,382}{40,595 + 75,193 - 5,382 - 37,915} = .4858 \\
 a &= \frac{40,595 - 5,382}{40,595 + 75,193 - 5,382} = .3189 \\
 b &= \frac{40,595}{40,595 + 75,193 - 37,915} = .5213 \\
 l &= \frac{40,595}{40,595 + 75,193} = .3506
 \end{aligned}$$

The results are in Table 3. We see that for Alabama the proportion part-time, already considerably higher than Illinois in 1959, had the greatest absolute increase over the succeeding ten year period. Yet the increase for Alabama was entirely due to the decline in the number of full-time farmers. In fact the number of part-time farmers declined also, and so contributed a relatively small negative component to the proportional change. Of the slightly lower absolute increase in the proportion part-time for Illinois, one-third is due to the increase in the number of part-time farmers, and two-thirds to the decline in full-time farmers. The components approach shows in a systematic way how the basis of change in a proportion may differ for different units. For both states considered here, the increase in the proportion part-time is primarily due to the numbers of full-time farmers leaving agriculture. Although Alabama continued to have the highest proportion part-time, and actually gained more in this proportion than Illinois over the decade, it was only Illinois that showed part of its proportional increase due to the increased prevalence of part-time farming.

For both states the interaction term is small relative to the other terms, in each case less than 10 percent of the magnitude of the smaller component. This term shows the difference between the estimate obtained for A by (1) above, and that which would have been obtained if only the

situation with B present, or the situation with B absent had been considered in calculating the A effect. For Alabama, the A effect for B present is $ab - b = .4858 - .5213 = .0355$, whereas the A effect for B absent is $a - 1 = .3189 - .3506 = .0317$. The interaction term is one-half the difference between these two estimates, or $-.0019$. The original A effect is $-.0336$, or $.0019$ from either $-.0355$ or $-.0317$. Consequently the interaction term may be seen here as a kind of indicator of the precision of the estimate of A, or following a similar development, of the estimate of B.

Following factorial analysis of variance procedures, computations may be simplified by means of matrix algebra. For the two component case:

$$1/2 \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} ab \\ a \\ b \\ 1 \end{pmatrix} = \begin{pmatrix} A \\ B \\ AB \end{pmatrix} \quad (5)$$

Two Components in Numerator and Two in Denominator

Suppose $\Delta\alpha$ above could be broken into two components $\Delta c + \Delta d$, and Δb could be broken into two components $\Delta e + \Delta f$. Then the basic change formula would be

$$\frac{X + \Delta c + \Delta d}{X + Y + \Delta c + \Delta d + \Delta e + \Delta f} - \frac{X}{X + Y} \quad (6)$$

with X and Y defined as before. We wish to partition this difference into additive C, D, E, and F effects.

The 2^3 factorial: For our first approach we define the C and D effects according to the 2^3 factorial having the three factors c, d, and cd, and the E and F effects according to the 2^3 factorial having the three factors e, f, and ef. Consistent with the previous development, define the treatment combination as follows:

$$cdef = \frac{X + \Delta c + \Delta d}{X + Y + \Delta c + \Delta d + \Delta e + \Delta f} \quad \text{which is the same as ab before}$$

$$cdf = \frac{X + \Delta c + \Delta d}{X + Y + \Delta c + \Delta d + \Delta f}$$

$$cde = \frac{X + \Delta c + \Delta d}{X + Y + \Delta c + \Delta d + \Delta e}$$

$$cd = \frac{X + \Delta c + \Delta d}{X + Y + \Delta c + \Delta d} \quad \text{which is the same as a before}$$

$$cef = \frac{X + \Delta c}{X + Y + \Delta c + \Delta e + \Delta f}$$

$$c = \frac{X + \Delta c}{X + Y + \Delta c}$$

$$def = \frac{X + \Delta d}{X + Y + \Delta d + \Delta e + \Delta f}$$

$$d = \frac{X + \Delta d}{X + Y + \Delta d}$$

$$ef = \frac{X}{X + Y + \Delta e + \Delta f} \quad \text{which is the same as b before}$$

$$f = \frac{X}{X + Y + \Delta f}$$

$$e = \frac{X}{X + Y + \Delta e}$$

$$1 = \frac{A}{A + B} \quad \text{as before.}$$

Let the C effect be the average of four differences between treatment combinations with c present and c absent: 1) when d, ef are present; 2) when d absent, ef present; 3) when d present, ef absent; 4) when d and ef are both absent:

$$C = 1/4[(cdef - def) + (cef - ef) + (cd - d) + (c - 1)] \quad (7)$$

Similarly,

$$D = 1/4[(cdef - cef) + (cef - ef) + (cd - c) + (d - 1)] \quad (8)$$

Then $C + D = 1/2[cdef - ef + cd - 1] = 1/2 [ab - b + a - 1] = A$

Let the E effect be the average of four differences between e present and absent: 1) when f and cd are present; 2) when f absent, cd present; 3) when f present, cd absent; and 4) when f and cd both absent:

$$E = 1/4 [(cdef - cdf) + (cde - cd) + (ef - f) + (e - 1)] \quad (9)$$

$$F = 1/4 [(cdef - cde) + (cdf - cd) + (ef - e) + (f - 1)] \quad (10)$$

$$\text{Then } E + F = 1/2 [cdef - cd + ef - 1] = 1/2 [ab - a + b - 1] = B \quad (12)$$

Finally $C + D + E + F = [cdef - 1] = A + B = [ab - 1]$, the basic change formula (6), as seen by adding up terms (7) through (10) above.

One may also define CD and EF interaction terms:

$$CD = 1/4 [(cdef - def) - (cef - ef) + (cd - d) - (c - 1)] \quad (11)$$

$$EF = 1/4 [(cdef - cdf) - (cde - cd) + (ef - f) - (e - 1)]$$

Again, these interaction terms could give an indication of the precision of these estimates, or the variation of the effect differences over the four circumstances considered.

The 2^4 factorial: The preceding approach provides a solution in which the four components add to the difference between the proportion at the first and second times, that is, the basic change formula. Yet in following the analysis of variance parallel, using a 2^4 design is not only more straightforward, but the effects would be averaged over two additional circumstances in each case, presumably yielding better estimates for the components.

In addition to the treatment combinations defined above, four additional combinations are required for the 2^4 model. They are:

$$cf = \frac{A + \Delta c}{A + B + \Delta c + \Delta f}$$

$$ce = \frac{A + \Delta c}{A + B + \Delta c + \Delta e}$$

$$df = \frac{A + \Delta d}{A + B + \Delta d + \Delta f}$$

$$de = \frac{A + \Delta d}{A + B + \Delta d + \Delta e}$$

With this number of terms, computation of the effects is best shown through matrix algebra. If we multiply the 15 by 16 design matrix by 1/8, and post multiply it by the 16 by 1 vector of treatment combinations, the result is a 15 by 1 vector of effects, including the four main effects, six two-way interactions, four three-way interactions, and one four-way interaction:

1/8

1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	cdef	C
1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	cdf	D
1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	cde	E
1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	cd	F
1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	cef	CD
1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	cf	CE
1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	ce	CF
1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	c	DE
1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	def	DF
1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	df	EF
1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	de	CDE
1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	d	CDF
1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	ef	CEF
1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	f	DEF
1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	e	CDEF
																1	

With the 2⁴ model, an E effect term, for example, would be:
1/8 [(cdef-cdf)+(cde-cd)+(cef-cf)+(ce-c)+(def-df)+(de-d)+(ef-f)+(e-1)]. (14)
This is an average across eight terms instead of four terms as before, with the first two and the last two terms here being those of the E effect for

the previous model. Thus the new E effect would equal the old E effect if together the middle four terms equaled the sum of the first two and last two terms. Note that for the first two terms c and d are present together, for the next two c is present but not d, for the next two d is present but not c, and for the last two neither c nor d are present. Thus a zero difference between the middle four terms and the first two and the last two together would indicate zero CDE interaction, since it would make no difference whether c and d were considered separately or together in calculating the E effect. From the matrix, the CDE interaction term is seen to be:

$$CDE = 1/8[(cdef-cdf)+(cde-cd)-(cef-cf)-(ce-c)-(def-df)-(de-d)+(ef-f)+(e-1)] \quad (15)$$

Consequently, the E effect, plus the CDE interaction equals the E effect on the earlier model. This can be seen easily by adding together the third and eleventh rows of the design matrix, yielding the 1 by 16 vector

$$(2 \ -2 \ 2 \ -2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2 \ -2 \ 2 \ -2)$$

Premultiplying this by one-eighth, and post multiplying by the 16 by 1 treatment combination vector results in the previous E effect

$$1/4 [(cdef - cdf) + (cde - cd) + (ef - f) + (e - 1)].$$

By rearranging terms, the same relations between the 2^3 and 2^4 models may be shown for the C, D, and F effects. Consequently, for the 2^4 model, the sum of the C, D, E, and F effects and the CDE, CDF, CEF, and DEF interactions equals the basic change formula, $(cdef - 1)$. Note that the sum of the first four and the eleventh through fourteenth rows of the design matrix is the 1 by 16 vector

$$(8 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -8)$$

which when multiplied by 1/8 and the treatment combination vector equals $(cdef - 1)$.

In a similar way, it is easy to show that the earlier CD and EF interaction terms (formulas 11 and 12) are equal to the 2^4 model (CD + CDEF) and (EF + CDEF) interaction terms, respectively.

A practical procedure, then, would be to use the 2^4 model to partition the basic change formula into the four main effects (C, D, E, F) and the four three-way interactions. If the four latter terms are uniformly small relative to the former four, one may choose to combine main effects with corresponding interactions, or in other words to use the 2^3 model developed previously, so that the main effects will uniquely add to the basic change formula.

Changing Age Structure--an example: That the population of the United States is "aging" is commonplace knowledge. One way to measure this process is to take the difference between the percent or proportion of the population over a given age for two time periods; thus the proportion of the U.S. population over 65 went from .092 to .099 between 1960 and 1970, for a difference of .007. From the approach developed so far, we know that this increment is probably due to the growth of the population over 65, offset in part by the growth of the population less than 65. Comparing different time periods or different areas should throw light on the aging process. Previous studies of different areas suggest the value of considering the demographic components of these changes as well, that is, migration and "natural increase". Thus, the areas with current high levels of the proportion over 65 and over, and increases in this proportion, include Sunbelt regions and parts of the Great Plains, with migration of older people assumed to be an important basis for increased proportions in the former areas and outmigration of young people an important basis for this situation in the latter area. (See Van Es and Bowling, 1978-9, for a path model of aging

which includes young and old migration for Illinois counties.) The basic 1960-70 data are given for Florida and South Dakota in Table 4. The net migration estimates by age are from Bowles, et al. (1975) with the other information derived from census sources. Note that "natural increase" here must include gains of population moving into the over 65 category, and corresponding losses to the under 65 category, as well as deaths and births. The treatment combinations, rounded to five places, are given in Table 5, followed by the main effects and interactions in Table 6.

The two-way interactions are all small relative to the main effects, although the effects appear to be less stable for Florida under different treatment combinations. Three of the six Florida two-way interactions are about 18 percent of the size of the smallest corresponding main effect, whereas for South Dakota only one interaction term exceeds five percent. The three-way interactions are all quite small, never exceeding one-half of one percent of the main effect with which they are combined to obtain effects for the 2^3 design. Consequently, using the 2^3 approach appears appropriate here.

A components analysis is summarized in Table 7 for the 1960-70 change in the proportion over 65 in Florida and South Dakota. Both states had more than 10 percent of their population over 65 at both censuses, well above the U.S. as a whole. Florida gained more than three percentage points, whereas South Dakota gained only one and one-half percentage points over the ten year period. What is more significant, however, is that the bases of these changes are quite different. Florida has gained considerable population through migration, and so it is not surprising that the aged migration component amounts to 5.5 percent. "Aging in place," that is, movement into the older age category minus deaths, adds another percent, but almost one-half

of this total gain is offset by changes in the denominator, with a two percentage point loss due to gains in net migration of younger people, and a one percentage point loss due to natural increase (in this case births, minus deaths, minus losses due to residents becoming 65 years old during the decade).

In South Dakota, on the other hand, there is virtually no gain due to the migration of the elderly, and indeed the largest positive component is due to the outmigration of younger people, followed by the aging in place component. The negative impact of younger natural increase is similar to that for Florida, but it is outweighed on the denominator side by younger net migration loss. Approximately 75 percent of the gain in proportion over 65 in South Dakota is due to gains to the older population (almost entirely gains in place) but 25 percent is due to decline in the younger population because net outmigration offsets natural increase.

Discussion

With this approach, the main effects are averages of the difference between presence and absence of a factor across different combinations of the other factors. Thus the other factors are "controlled" as they are in the analysis of variance. Each treatment combination here may be considered to represent what the proportion would be, given the presence only of the factors indicated. In the previous example, cdf is what the proportion over 65 would be, if there had been no net migration of persons 0-64 over the decade, but all other changes were as observed. Then $cdef - cdf$ is the effect of net migration 0-64 on the proportion over 65 with all other changes observed. For the 2^4 model, however, the E effect is the average of eight treatment differences (presence minus absence of e) over all combinations of presence or absence of the other factors:

- | | | |
|----|---------------|-------------------------------------|
| 1. | (cdef - cdf): | all other factors (c, d, f) present |
| 2. | (cde - cd): | c, d present |
| 3. | (cef - cf): | c, f present |
| 4. | (ce - c): | c present |
| 5. | (def - df): | d, f present |
| 6. | (de - d): | d present |
| 7. | (ef - f): | f present |
| 8. | (e - 1): | no other factors present |

Our results show that for Florida, the effect of migration of those under 65 is to lower the percent over 65 by 2.11, controlling for changes in natural increase of the under 65 and the 65 and over groups, and net migration of those 65 and over.

In a recent paper, Larry Long (1975) presents a related method to indicate how the proportion black changes for large cities. He calculated two treatment combinations as follows:

1. The proportion nonwhite at the end of the period if there had been no nonwhite or white migration but with natural increase (df in the notation of this paper; and
 2. The proportion nonwhite with no white migration but with nonwhite migration and natural increase, and white natural increase (cdf here).
- Then with the proportion nonwhite at the end (cdef) and at the beginning (1) of the period his components equation may be given as follows:

$$cdef - 1 = (cdef - cdf) + (cdf - df) + (df - 1).$$

The first term on the right is his effect of white migration (E), the second the effect of nonwhite migration (C) and the third the effect of natural increase of both (D + F). The latter term is interpreted as the effect of higher natural increase of nonwhite since the effect of a

positive natural increase for whites in the denominator is negative, and nonwhite natural increase is always higher than white.

Results are presented in his paper for 11 cities. Table 8 presents a comparison of the two methods for New York City, calculated using data in Bowles and Tarver (1965). With the method developed here the 2^3 model is used, since the interaction terms are in size never more than 12 percent of corresponding main effects, and the sum of the three-way interactions of $-.00015$. As expected, the results of the two methods are similar. Even though the interaction terms are small, however, the effect of nonwhite net migration is four percentage points smaller and white net migration and the natural increase differential are about two percentage points more with Long's method. Presumably the present method gives better estimates of the effects, since differences are averaged over combinations of other effects, whereas Long simply takes one of these terms for each of his effects. Although the approach here provides separate natural increase effects, Long could have done this easily, by adding either the d or f treatment combination, to partition the $df - 1$ effect. This illustrates however the non-uniqueness of the approach, since $(f - 1)$ and $(df - d)$ could either one be the natural increase effect for whites, depending upon whether white natural increase is introduced first or second. One of the six alternate developments of his initial model is as follows:¹

$$cdef - 1 = (cdef - ce) + (ce - e) + (e - 1).$$

Again, the $(cdef - ce)$ term could be made equal to $(cdef - ced) + (ced - ce)$ to yield a white and a nonwhite natural increase term respectively. All in all, the ANOVA-based model presented here should be preferable to that developed by Long, or to any alternative single-term effect models. It must

be said, however, that his model has the advantage of ease of presentation and substantive interpretation. Yet for most studies, if one could expect differences as large as that found here for the New York city example, which was under conditions of negligible interaction, most prudent researchers should wish at least to compare the results of any single-term effect approach with those for this ANOVA-based model.

Possible applications of the two or four component models developed in this paper are not limited to research on occupational change and migration. Another possible demographic use is in the study of urbanization. Kingsley Davis (1972) and others have given attention to the fact that change in level of urbanization, as measured by proportion urban is not synonymous with urban population growth. A four component model might break the change over time in percent urban into that attributed to large and small urban places, and into two denominator terms for rural villages and the open country.

There is a wide potential for nondemographic applications as well, where time series are available with neither A, B, nor $A + B$ fixed in advance. (The latter circumstance would include considering change in the proportion across two surveys with sample sizes, and hence $A + B$, predetermined.) Two examples relating to criminology will be mentioned. According to the FBI Uniform Crime Reports (1979), the percent of arrests for crimes classified as violent, as contrasted with crimes against property, increased from 17.6 to 19.1 between 1970 and 1979. This 1.5 percentage point increase is a composite of 5.9 for the effect of increase in arrests for violent crimes on the percent violent, offset by a 4.4 for the increase in arrests for property crimes in the denominator. Allowing for differences by sex with the 2^3 model, the male violent crime component is 5.2 and female 0.7, whereas the male property component is -2.9 and female -1.5. Consequently we can say that

females contributed only a small proportion of the violent crime effect on the change in the percentage of arrests that are for violent crimes, but one-third of the offsetting property crime effect on this proportion through the increase in arrests for property crimes over the decade. Looking at the contribution of violent crimes and property crimes for males and females separately, we see further that together males contribute +2.3 to the increase in percentages of arrests for violent crimes (5.2 - 2.9), whereas females would lower the increase by 0.8, since arrests for property crimes have increased more rapidly than for violent crimes for females (0.7 - 1.5).

Between 1940 and 1975, the percentage of homicide victims who were black dropped from 58.7 to 48.5, or 6.2 percentage points (U.S. Bureau of the Census, 1978). Controlling (as is done by this method) for changes in white homicides, the proportion of homicides that are black would have increased 18.8 percent rather than declined. The decline then is due to the even greater increase in the white homicides, which if black homicides were controlled, would have resulted in a decline of 25 percentage points. Again considering males and females in a 2^3 model, the black male component is 15.0, female 3.8, white male -17.9 and female -7.1. Though separate white and black male effects are considerably larger than those for females, note that the overall male contribution to the decline in the percentage of homicides black is -2.9 (15 - 17.9) whereas the female contribution is -3.3, or more than one half of the percentage point decline. Female effects are lower, but the female black-white differential is higher, in its influence on the percent of homicide victims who are black.

Finally, it is important to recognize that this method is not a substitute for, nor is it replaced by, a systematic comparison of rates. In studying migration of the elderly, or of blacks and whites, or in considering other

problems similar to the examples given here, one would usually wish to include an examination of separate rates of change. For example, a comparison of rates of net migration and natural increase of those over and under 65 in Florida and South Dakota would provide valuable information. Rates would be sufficient for an analysis, however, only if there is no interest in how these subgroup changes contribute to the basic proportional change, as in the components equation.

The study of proportional change is an important task. As is widely recognized today, it is necessary to understand structural as well as simple growth or decline, with structural change occurring when an aggregate consists of interrelated parts, and there are changes in the proportional relations between the parts (Boulding, 1953). The consequences of such structural changes as the aging of a population, or the increase in the proportion that belong to a minority, are as yet too little understood, and this approach provides a first step by explicating the basis of such changes, which may be subtle and complex. I hope that this method will contribute to further advances in knowledge about social change.

Table 1

TREATMENT COMBINATIONS

		Changes in full-time	
		Yes	No
Changes in	Yes	ab	a
part-time	No	b	1

Table 2

PART-TIME FARMING DATA

State and Farmer Type	1959	1969	difference
Alabama:			
Part-time	40,595	35,213	- 5,382
Full-time	75,193	37,278	-37,915
All farmers	115,788	72,491	
Illinois:			
Part-time	33,815	41,553	- 7,738
Full-time	120,829	82,012	-38,817
All farmers	154,644	123,565	

SOURCE: U.S. Bureau of the Census, 1961 and 1972.

Table 3

PROPORTION PART-TIME AND EFFECTS

	Proportion Part-Time			Effects			
	1969	-	1959 = difference =	A	+	B	: AB
Alabama	.4858		.3506	.1352	-.0336	.1688	-.0019
Illinois	.3363		.2187	.1176	.0408	.0768	.0036

Table 4

DATA ON MIGRATION BY AGE

	Florida	South Dakota
Population 65 and over in 1960	553,127	71,513
Population 0-64 in 1960	4,398,424	608,996
Net migration 1960-70, population 65 and over as of 1970	371,985	257
Natural increase 1960-70, population 65 and over in 1970	60,410	8,504
Net migration, 1960-70, population 0-64 as of 1970	959,103	-94,199
Natural increase 1960-70, population 0-64 in 1970	448,338	71,183

SOURCE: Bowles, Beale and Lee, 1975.

Table 5

TREATMENT COMBINATIONS, FLORIDA AND SOUTH DAKOTA

Treatment Combinations	Florida	South Dakota
cdef	.14511	.12049
cdf	.16898	.10556
cde	.15537	.13490
cd	.18305	.11646
cef	.13744	.10911
cf	.16028	.09545
ce	.14725	.12236
c	.17378	.10543
def	.09558	.12015
df	.11236	.10526
de	.10275	.13452
d	.12241	.11613
ef	.08698	.10877
f	.10243	.09514
e	.09358	.12197
l	.11171	.10509

Table 6
MAIN EFFECTS AND INTERACTIONS, FLORIDA AND SOUTH DAKOTA

Effects and Interactions	Florida	South Dakota
C	.05543	.00034
D	.00902	.01127
E	-.02137	.01597
F	-.01009	-.01212
CD	-.00058	.00000
CE	-.00386	.00002
CF	-.00182	-.00002
DE	-.00063	.00069
DF	-.00030	-.00052
EF	.00163	-.00169
CDE	.00009	.00000
CDF	.00004	.00000
CEF	.00024	.00000
DEF	.00004	-.00006
CDEF	-.00001	.00000

Table 7

COMPONENTS OF CHANGE IN PROPORTION, FLORIDA AND SOUTH DAKOTA

Components	Florida		South Dakota	
	2 ⁴ model	2 ³ model	2 ⁴ model	2 ³ model
C (net migration 65 and over)	.0554	.0555	.0003	.0003
D (natural increase 65 and over)	.0090	.0091	.0113	.0113
(Change in 65 and over)		(.0646)		(.0116)
E (net migration 0-64)	-.0213	-.0211	.0160	.0160
F (natural increase 0-64)	-.0101	-.0101	-.0121	-.0122
Change in 0-64)		(-.0312)		(.0038)
Three-way interaction	.0004	---	-.0001	
<u>Change in Proportion (cdef - 1)</u>	<u>.0334</u>	<u>.0334</u>	<u>.0154</u>	<u>.0154</u>
Proportion 65+ in 1970 (cdef)	.1451	.1451	.1205	.1205
Proportion 65+ in 1960 (1)	.1117	.1117	.1051	.1051

Table 8

COMPONENTS OF CHANGE IN PROPORTION NONWHITE, NEW YORK CITY, 1950-60

Components	Fuguitt 2 ³		Long	
	Effects	% Dist	Effects	% Dist
C (net migration nonwhite)	.0190	39.3	.0171	35.3
D (natural increase nonwhite)	.0220			
E (net migration white)	.0156	32.2	.0167	34.5
F (Natural increase white)	-.0082			
D + F (natural increase nonwhite and white)	.0138	28.5	.0146	30.2
Total Main Effects	.0484	100.0	.0484	100.0

SOURCE: Bowles and Tarver, 1965

FOOTNOTE

¹These six developments include four different estimates of the E, C, and D + F effects. Across the different estimates, the percentages, as in Table 8, have the following ranges: E--29.1 to 34.5; C--35.3 to 44.0; D + F--26.4 to 31.4. As would be expected, the average of these percentages for a given effect is very close (within .2 of a percent) to the corresponding percentage in the 2^3 model.

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